Remarks, hints and tricks on writing theses

Bachelor's & Master's Thesis Seminar ST 2020

Franz Gmeineder

May 07, 2020

So you are approaching your graduation thesis ...

... well done that you made it to this point!

$\longrightarrow$ time to top 3 or 5 years off with a remarkable thesis

$\longrightarrow$ What are the objectives?

Your thesis should ...

- demonstrate your ability to absorb mathematical ideas ...
- and to present them in a way such that

  a fellow student with your preknowledge prior to the work on the thesis' topic can follow your explanations, and
- demonstrate your ability to work scientifically.

What does that mean?

Let us rather start with: What not?

Think of truly bad lecture notes:

- **No motivation of what follows.**

$\rightsquigarrow$ You will have no clue of **why** the sequel is interesting and why should actually be eager to delve into the material.

- **Definition, theorem, proof-style, no connections.**

$\rightsquigarrow$ You will have no clue why a certain definition is posed until you work yourself through the subsequent theorems.

$\rightsquigarrow$ You, the reader, must find the **greater picture** of the content on your own.

$\rightsquigarrow$ Sometimes it is difficult to extract why the theory provided so far is useful at all.

- **Messy and/or useless referencing.**

$\rightsquigarrow$ Inside: No use of the LaTex referencing packages are made or if so, the references do not fit.

$\rightsquigarrow$ Outside: Later ...

Let us rather start with: What not?

Think of truly bad lecture notes:

- **No motivation of what follows.**

⇝ You will have no clue of **why** the sequel is interesting and why should actually be eager to delve into the material.

- **Definition, theorem, proof-style, no connections.**

⇝ You will have no clue why a certain definition is posed until you work yourself through the subsequent theorems.

⇝ You, the reader, must find the **greater picture** of the content on your own.

⇝ Sometimes it is difficult to extract why the theory provided so far is useful at all.

- **Messy and/or useless referencing.**

⇝ Inside: No use of the LaTex referencing packages are made or if so, the references do not fit.

⇝ Outside: Later ...

Let us rather start with: What not?

Think of truly bad lecture notes:

- **No motivation of what follows.**

⇝ You will have no clue of **why** the sequel is interesting and why should actually be eager to delve into the material.

- **Definition, theorem, proof-style, no connections.**

⇝ You will have no clue why a certain definition is posed until you work yourself through the subsequent theorems.

⇝ You, the reader, must find the **greater picture** of the content on your own.

⇝ Sometimes it is difficult to extract why the theory provided so far is useful at all.

- **Messy and/or useless referencing.**

⇝ Inside: No use of the LaTex referencing packages are made or if so, the references do not fit.

⇝ Outside: Later ...

Let us rather start with: What not?

Think of truly bad lecture notes:

- **No motivation of what follows.**

$\rightsquigarrow$ You will have no clue of **why** the sequel is interesting and why should actually be eager to delve into the material.

- **Definition, theorem, proof-style, no connections.**

$\rightsquigarrow$ You will have no clue why a certain definition is posed until you work yourself through the subsequent theorems.

$\rightsquigarrow$ You, the reader, must find the **greater picture** of the content on your own.

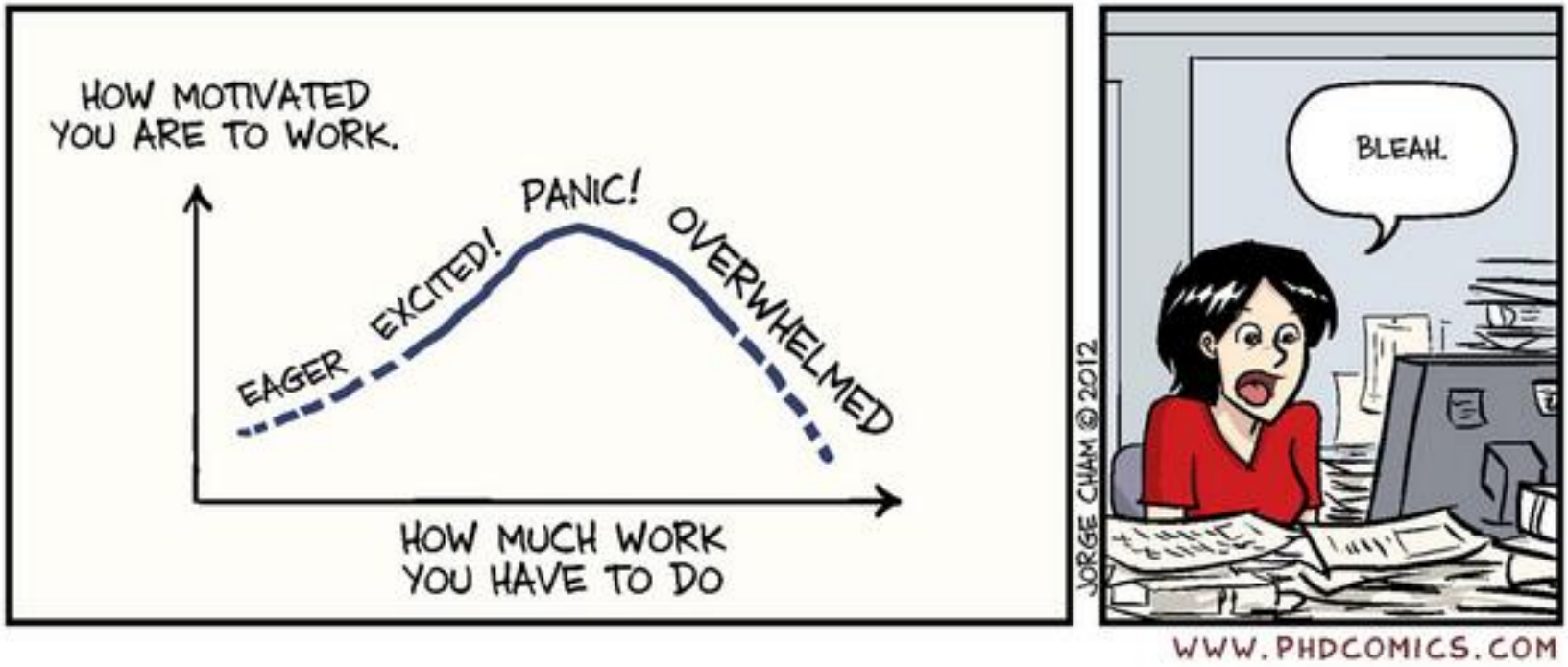
$\rightsquigarrow$ Sometimes it is difficult to extract why the theory provided so far is useful at all.

- **Messy and/or useless referencing.**

$\rightsquigarrow$ Inside: No use of the LaTex referencing packages are made or if so, the references do not fit.

$\rightsquigarrow$ Outside: Later ...

Timeframes and deadlines



- Preparations and preliminary writings ($\approx$ 1-2 months)
- Thesis (morally) done $\approx$ 1 month before final submission
- **Do not underestimate the amount** of the final fine-tuning!

Structure of the presentation

- **Section 1:** Aims and scope
- **Section 2:** Structuring your document.
- **Section 3:** The common thread.
- **Section 4:** Style.
- **Section 5:** Referencing.

A possible thesis structure

1. **Introduction**
   - ⇝ Motivation, aims, tools to arrive there; what happens in your thesis and where. Contextualisation.
2. **Preliminaries**
   - ⇝ Notation, auxiliary results, background material.
3. **Main part/proof the main theorem**
   - ⇝ go beyond lecture notes: Explain carefully your strategy and how the results are connected. Are there underlying heuristics?
4. **Conclusion**
   - ⇝ What has been achieved? What could come next? Outlook ...
5. **References**
   - ⇝ Concise list of the references utilised in the main text.

Guiding the reader through your document

Throughout, it must be clear

- **what** you are doing,
- **why** you are doing it (motivation – why is this of interest at all? What is the context of this particular question?), and
- **how** you are doing it. Which tools do you provide to tackle a certain question?
- How are these tools used – and in which order?

This structuring particularly applies to

- the **introduction** of your thesis ($\approx$ 5–10% of the total page count).
- the introductory subsection of each chapter.

$\rightsquigarrow$ equally important in longer texts with a plenty of auxiliary results that are required towards a major theorem

Some hints

A guiding principle

A fellow student of yours who has exactly your preknowledge at the beginning of your work on the thesis should not have intense problems to follow.

... and complementing this idea, make sure to ...

... have a consistency check

- Suppose that you use a notion $\mathfrak{A}$.

$\rightsquigarrow$ Had you been familiar with this notion prior to your work on your thesis?

$\rightsquigarrow$ Is the notion introduced before its first effective appearance?

... play dumb!

You become more and more an expert in your thesis topic – play dumb:

- Could you follow your own arguments? (Let some time pass between writing and reading...)
- Would you enjoy reading your own text? ($\longrightarrow$ style)

Some hints

A guiding principle

A fellow student of yours who has exactly your preknowledge at the beginning of your work on the thesis should not have intense problems to follow.

... and complementing this idea, make sure to ...

... have a consistency check

- Suppose that you use a notion $\mathfrak{A}$.

$\rightsquigarrow$ Had you been familiar with this notion prior to your work on your thesis?

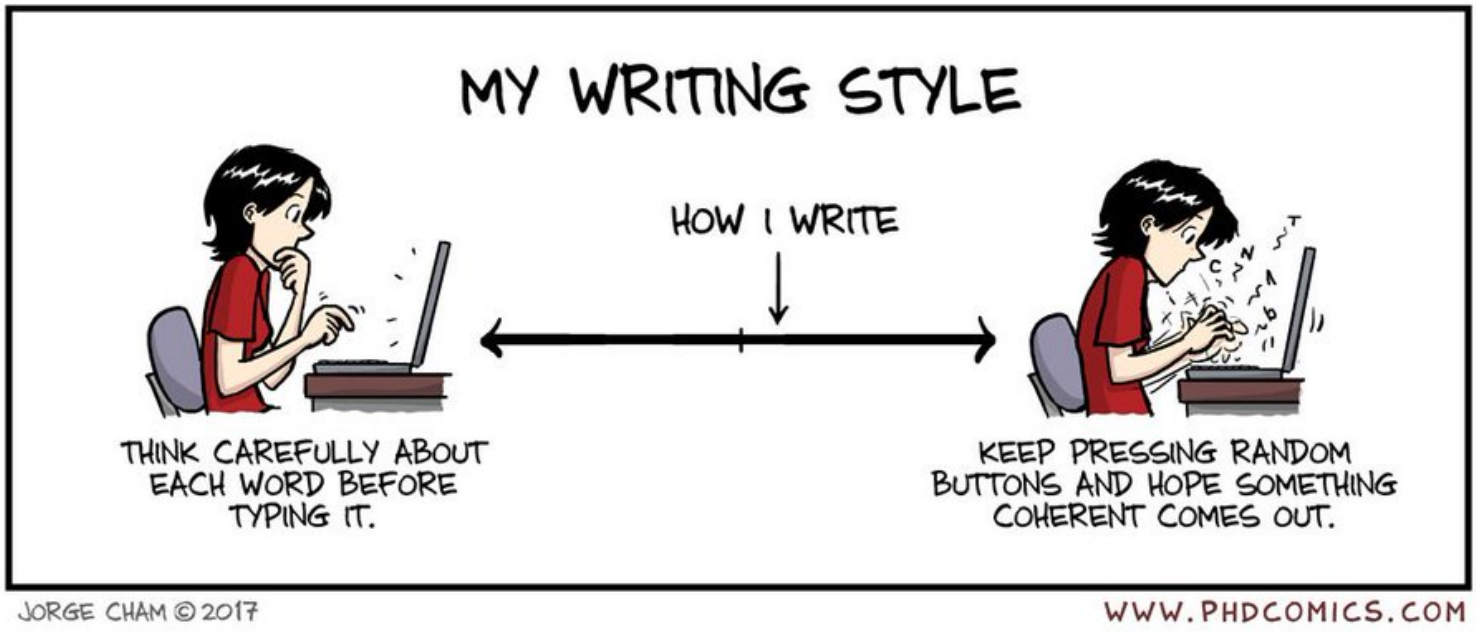
$\rightsquigarrow$ Is the notion introduced before its first effective appearance?

... play dumb!

You become more and more an expert in your thesis topic – play dumb:

- Could you follow your own arguments? (Let some time pass between writing and reading...)
- Would you enjoy reading your own text? ($\longrightarrow$ style)

Style



- **Developing your own (writing) style** is another central objective
- Getting inspired by others $\longleftrightarrow$ adapt the styles of others too much (unnatural writing)

How to avoid being too close to others

A possible strategy

- Never work directly from a paper or a book. Produce notes of a text that you are currently considering.
- Do not formulate too much (on these notes).
- Work directly from these notes.

$\rightsquigarrow$ Proceeding in this way, you immediately see whether there are any comprehension problems of yours.

Paraphrasing

Paraphrasing others without clearly naming it/referencing is very close to plagiarism. So avoid it ;)

Some more remarks

Would you enjoy reading your own writings?

- Avoid too long sentences. After a first writing, split.
- Check: Are your sentences proper sentences?
- As to mathematics: Are the assumptions clear?

Example (Some examples how it should not be done)

- *This follows from (2.4). And the isoperimetric inequality.*
- **Theorem 3.10.** *The Fourier transform is an isomorphism.*

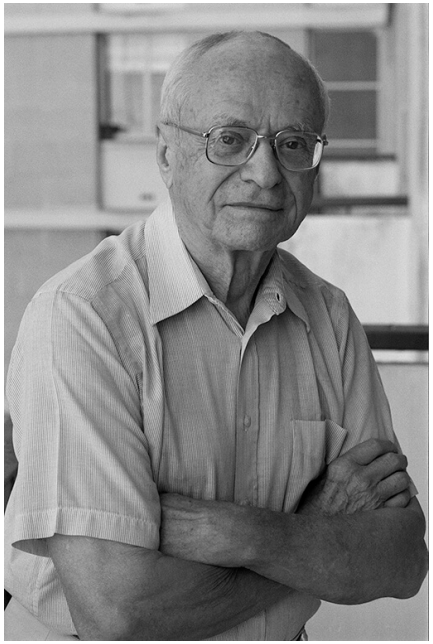
Safety measures

- Ask your fellow students **for overly critical feedback**. Do they like your writings and understand what you are aiming at?

$\rightsquigarrow$ **Critical feedback** sometimes difficult, in particular among friends ... :)

Some hints from the master

A very instructive video that is certainly worth your while by J.-P. Serre:



'How to write mathematics badly' – find the video on YouTube:

`https://www.youtube.com/watch?v=ECQyFzzBHlo`

On (mathematical) exposition and typesetting I

- Good grammar and orthography are a **must**.

$\rightsquigarrow$ When writing in English, make sure to follow the rules of the English language! (adverbs, avoid false friends,...)

And as to mathematics – here is a quick list:

Formulas within the text

To emphasize formulas, it is often a good idea to spend a new line (and make use of the `align` or `equation` environment:

Unfavourable: *Since the integral* $\int_{\mathbb{R}^n} \frac{\sqrt{1+|\xi|^2}}{\sqrt{1+|\xi|^{10n}}}\,\mathrm{d}\xi$ *exists, the right-hand side quantity of (2.30) can be bounded by ...*

Better: *Since the integral*

$$\int_{\mathbb{R}^n} \frac{\sqrt{1+|\xi|^2}}{\sqrt{1+|\xi|^{10n}}}\,\mathrm{d}\xi$$

exists, the right-hand side quantity of (2.30) can be bounded by ...

On (mathematical) exposition and typesetting I

- Good grammar and orthography are a **must**.

$\rightsquigarrow$ When writing in English, make sure to follow the rules of the English language! (adverbs, avoid false friends,...)

And as to mathematics – here is a quick list:

Formulas within the text

To emphasize formulas, it is often a good idea to spend a new line (and make use of the `align` or `equation` environment:

Unfavourable: *Since the integral* $\int_{\mathbb{R}^n} \frac{\sqrt{1+|\xi|^2}}{\sqrt{1+|\xi|^{10n}}}\,\mathrm{d}\xi$ *exists, the right-hand side quantity of (2.30) can be bounded by ...*

Better: *Since the integral*

$$\int_{\mathbb{R}^n} \frac{\sqrt{1+|\xi|^2}}{\sqrt{1+|\xi|^{10n}}}\,\mathrm{d}\xi$$

exists, the right-hand side quantity of (2.30) can be bounded by ...

On (mathematical) exposition and typesetting II

Formulas in math mode!

When talking about formulas, make sure to use the math mode (`$...$`).

Write full sentences!

You are not writing lecture notes but a **coherent text**. Make sure that every single sentence has a subject, verb and object.
Unfavourable: *(...). Therefore, (3.12).*
Better: *(...). Therefore, (3.12) follows.*

Sentences do end with a dot ...

... even if the very last part is a formula.
Example: *This allows us to conclude that the very left-hand term of (3.2) can be estimated by*

$$\int_{\mathbb{R}^n} \frac{|f(x)|^2}{\sqrt{1+|x|^2}}\,\mathrm{d}x.$$

Note the red dot :)

On (mathematical) exposition and typesetting II

Formulas in math mode!

When talking about formulas, make sure to use the math mode (`$...$`).

Write full sentences!

You are not writing lecture notes but a **coherent text**. Make sure that every single sentence has a subject, verb and object.
Unfavourable: *(...). Therefore, (3.12).*
Better: *(...). Therefore, (3.12) follows.*

Sentences do end with a dot ...

... even if the very last part is a formula.
Example: *This allows us to conclude that the very left-hand term of (3.2) can be estimated by*

$$\int_{\mathbb{R}^n} \frac{|f(x)|^2}{\sqrt{1+|x|^2}}\,\mathrm{d}x.$$

Note the red dot :)

On (mathematical) exposition and typesetting II

Formulas in math mode!

When talking about formulas, make sure to use the math mode (`$...$`).

Write full sentences!

You are not writing lecture notes but a **coherent text**. Make sure that every single sentence has a subject, verb and object.
Unfavourable: *(...). Therefore, (3.12).*
Better: *(...). Therefore, (3.12) follows.*

Sentences do end with a dot ...

... even if the very last part is a formula.
Example: *This allows us to conclude that the very left-hand term of (3.2) can be estimated by*

$$\int_{\mathbb{R}^n} \frac{|f(x)|^2}{\sqrt{1+|x|^2}}\,\mathrm{d}x.$$

Note the red dot :)

And once again ...

- A mathematical idea/theory can be absorbed way faster provided the structure and ideas are clear.
- Theorems sometimes have a history ... prove that you made your way through the literature and contextualise!

$\rightsquigarrow$ For both of the previous items: Are there any heuristics? Arguments, which are 'intuitively' clear?

$\rightsquigarrow$ Sometimes, pictures help! `TikZ` is your friend :)

And once again ... on the assumptions

- Theorems without assumptions are not worth anything.
- Specify **standing assumptions**: *Throughout this section, we tacitly assume $\mathcal{H}$ to be a separable Hilbert space.*

$\rightsquigarrow$ No need to mention this in every single statement.

And once again ...

- A mathematical idea/theory can be absorbed way faster provided the structure and ideas are clear.
- Theorems sometimes have a history ... prove that you made your way through the literature and contextualise!

$\rightsquigarrow$ For both of the previous items: Are there any heuristics? Arguments, which are 'intuitively' clear?

$\rightsquigarrow$ Sometimes, pictures help! `TikZ` is your friend :)

And once again ... on the assumptions

- Theorems without assumptions are not worth anything.
- Specify **standing assumptions**: *Throughout this section, we tacitly assume $\mathcal{H}$ to be a separable Hilbert space.*

$\rightsquigarrow$ No need to mention this in every single statement.

Referencing – or: Making your work transparent

Example (Useless references)

'*..., as can be easily deduced from* [JD07]'

> [JD07] Doe, John: *Lectures on the empty set.* Lecture Notes, University of Invisibility, 2007, 1340 pp.. Retrieved from `www.randomjohndoe.uk` on May 05, 2020, 01:30 am.

Problems:

- Unfortunately, Prof. Doe removed his lecture notes on May 05, 2020, 01:31 am.
  - ⇝ How should anyone be able to get this reference?
- Suppose that Prof. Doe would not have removed his notes.
  - ⇝ Is the reader really required to go through 1340 pages to finally find that the claim does follow easily from Thm. 5.10.42 on page 902 of [JD07]?
  - ⇝ Even worse: No single line from [JD07] allows to deduce any statement of your thesis easily ...

Referencing – or: Making your work transparent

Example (Useless references)

'*..., as can be easily deduced from* [JD07]'

> [JD07] Doe, John: *Lectures on the empty set.* Lecture Notes, University of Invisibility, 2007, 1340 pp.. Retrieved from `www.randomjohndoe.uk` on May 05, 2020, 01:30 am.

Problems:

- Unfortunately, Prof. Doe removed his lecture notes on May 05, 2020, 01:31 am.

⇝ How should anyone be able to get this reference?

- Suppose that Prof. Doe would not have removed his notes.

⇝ Is the reader really required to go through 1340 pages to finally find that the claim does follow easily from Thm. 5.10.42 on page 902 of [JD07]?

⇝ Even worse: No single line from [JD07] allows to deduce any statement of your thesis easily ...

Referencing – or: Making your work transparent

Example (Useless references)

'*..., as can be easily deduced from* [JD07]'

> [JD07] Doe, John: *Lectures on the empty set.* Lecture Notes, University of Invisibility, 2007, 1340 pp.. Retrieved from `www.randomjohndoe.uk` on May 05, 2020, 01:30 am.

Problems:

- Unfortunately, Prof. Doe removed his lecture notes on May 05, 2020, 01:31 am.

⇝ How should anyone be able to get this reference?

- Suppose that Prof. Doe would not have removed his notes.

⇝ Is the reader really required to go through 1340 pages to finally find that the claim does follow easily from Thm. 5.10.42 on page 902 of [JD07]?

⇝ Even worse: No single line from [JD07] allows to deduce any statement of your thesis easily ...

In conclusion: In view of outside references, ...

- make sure that your references are precisely addressed and traceable.
- contain all the requisite bibliographical information:
  - Author(s), article name, journal, volume, year, etc.
  - Author(s), book name, publishing house, series name, volume, etc.

$\longrightarrow$ Here, the `BibTex`-environment makes life easy!

$\longrightarrow$ MathSciNet helps you to find the correct references.

Referencing is a serious matter!

Whenever a quote, graphic or anything else **is not your own outcome** – give references. If this is overburdening (and, e.g., each result of a chapter is from [JD07]), specify this at the beginning of the chapter.

Figures

For figures – and should they rather illustrate a topic than give the decisive explanation – you may refer to webpages (as long the author etc. are named).

References inside your document

- LaTex provides you with a plenty of packages that help to refer to previous or later points in your text (e.g., the `hyperref` package).
- Make sure that these references fit.
- These 'refs' should be visible (by default in LaTex).

On good practice

Make sure that the reader is not confused by your referencing.

Example: *As can be extracted from* (3.10), ...

Problematic: Formula (3.10) ranges over 14 lines. What do you refer to explicitely?

$\rightsquigarrow$ Strike a balance – sometimes, it is not sensible to number each line of a 14 lines formula (use the `split`-environment). But if you refer to some particular line very often, this might be not so favourable.

References and more reading

- Graphic on slide 4: J. Cham, PhD comics: *Piled higher and deeper*. http://edjowriwe.weebly.com/blog/motivation, originally from http://phdcomics.com
- Graphic on slide 9: J. Cham, PhD comics: http://phdcomics.com
- Graphic on slide 12: https://www.heidelberg-laureate-forum.org/laureate/jean-pierre-serre.html

Some more ideas are gathered here:

https://fxgmeineder.wixsite.com/meinewebsite/on-theses-and-presentations

Here you also find this presentation and a small essay on the topic.

Many thanks for your attention!