Theses

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Some remarks and hints on writing theses

This paper collects some (hopefully) useful hints and suggestions on writing theses. Please note that these are merely *suggestions* and no strict guidelines. The document shall grow as time evolves. If you have any suggestions, please send me an e-mail via

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1 Preparations

Before you start writing, you will be given (most of) the requisite background material for your thesis. Based on your choice of topic, we will have a chat about your preknowledge. This helps to find out *what you have to work yourself through to understand the main references*. Of course, you might realise some gaps while working on the main text (which is not an issue, anyway) – just fill them along the way. When going into the background material for your thesis, make sure that you identify the core points your thesis will be about. Understand why these points are interesting and how they are approached.

In my opinion, the *general goal* of Bachelor's or Master's theses is

- to delve into a mathematical topic, understand it, and
- to present it in a way such that your fellow students could understand it as well.

Mathematical papers are often written for other experts in the field. As such, many background facts are omitted (actually, they are assumed to be known by the expert reader). Collect such results and give careful justification. As to the second item from above, if your thesis is mostly bookwork, then a central objective is to present the material in a comprehensible way. Thus, in a first draft, give *all* the details that seem important to

you. It might be that we agree to remove some later on, but for the first draft, *include them.*

In general, it is worth making notes on the material and then directly work and write from these notes. This helps you to check whether the underlying mechanisms and key points are clear to you. Moreover, it helps you to not stick to the original texts and to write the thesis in your own words – also see Section 4 below.

For Bachelor students: Needless to say, your thesis is required to be typeset via Latex – other formats, such as 'Word'-documents, will not be accepted. If you are not familiar with producing texts in Latex, it might take you some while, but it is worth it: Tex provides you with an astonishing wealth of beautiful ways to create text documents. Learning Tex is simply another skill you acquire while working on your thesis. Moreover, you shall be well-prepared for your Master's thesis, equally being produced in the Tex-format. An introduction to Tex is available here:

https://latex.tugraz.at/latex/tutorial.

2 Length and format

Personally, I do not care so much about precise rules on how a thesis is formatted. However, it should be well readable and so it is favorable to have

- margin width of $3-4$ cm,
- font size 10 or 11 pt.

For Bachelor students, the thesis length might vary between 25 and 50 pages (maximum). For Master students, the thesis length might range between 50 and 100 pages. This strictly depends on whether you present some research of your own or not. Personally, I require pure bookwork theses to be longer.

3 Structure of your document

Your thesis should contain an

- *abstract*: Here you concisely summarise what your thesis is about. The abstract should never exceed one single page (rather half a page). Give the most important formulas only (if at all), and do not get lost in detail here.
- *table of contents*: Working with LaTex, this is done almost automatically. There are some options of how fine-tuned your table of contents is (the topdepth-environment), see here for more detail:

  https://www.latex-tutorial.com/tutorials/table-of-contents/.

- *introduction*: Here you should introduce the topic covered in your thesis. It is valuable to give good motivation of the theme you write about – so, for instance, start with an explicit problem that you aim to solve. What tools are provided in

the thesis to tackle this problem? In doing so, which other problems arise? Try to provide the reader with a *red line*: How are the single issues connected?

Present your main theorems (not minor results) in a crisp manner. Clarify why these results are interesting, non-trivial and give a rough outline of how they are approached. Mention where they are proved, and how they are utilised to tackle the problem you rised at the begininning of the introduction.

Finally, conclude the introduction with a quick *organisation of the thesis*: Which chapters are there? What are the main contents? Confine to a single sentence for each chapter.

Regarding the size of the introduction, a rule of thumb is that it should have a length of 10% of the main text.

- *section/chapter on preliminaries*: Here you fix notation and gather auxiliary definitions and results that you shall refer to in the main part of your thesis. Note that, depending on the specific topic, you might also choose to include the preliminaries into a different chapter. However, I find it worthwile having a concise blackbox for all of what follows.
- *main part*: See Section 4 below.
- *conclusion* – if you wish: Summarise what has been achieved in your thesis, and relate these achievements to the issues you explained in the introduction.
- *a bibliography/references section*: Here you gather the literature you quote throughout the main text. It is expected that you use BibTex or other bibliographical environments.

Have a look at my webpage, where some of the theses I supervised are linked and get yourself some inspiration.

4 Some notes on style

One of the many objectives of writing a thesis is to develop a coherent style to communicate mathematical ideas. There are no universal rules, and everybody writes somewhat differently. Mathematical texts usually do not require utmost oracy, but there are differences indeed. It is important to not copy a certain author's style – find your own style. If you have not written a mathematical text before, you are advised

- to delve into some preliminary material at an early stage of your project,
- and to tex 3–5 pages. Check carefully whether you *would enjoy to read your own text*. Make notes on what could be improved and modify your exposition.

If you reproduce a proof from a paper or a textbook, give the proof in your own words – otherwise you come very close to 'copy and paste'. To avoid such issues, work yourself through the proof and make notes on your own. Then write down the proof based on *these notes* (and not the particular reference). If you take a passage from a particular reference verbatim, you **must** declare it as a quotation. Sometimes it might happen that a reference is in a different language than that of your theses. If you translate a given

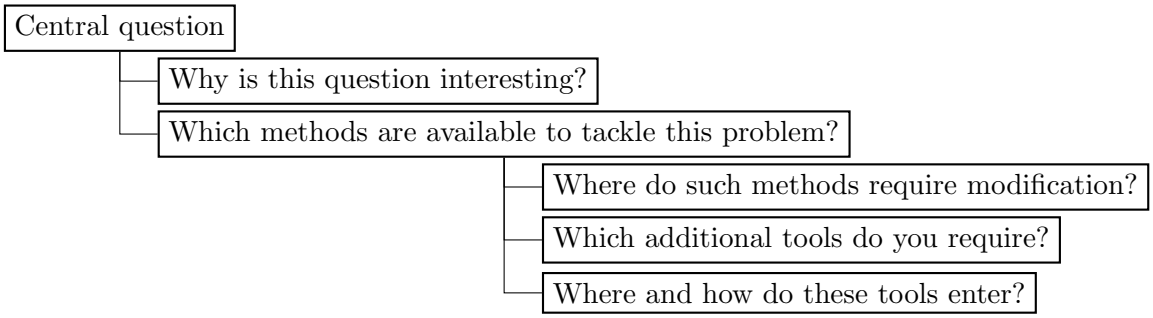

Figure 1: One way to provide the reader with a red line, starting from the central question (of the thesis or a single chapter).

reference, you must equally declare your translation as such.

Throughout your document, it must be clear

what you do, why you do it and how you do it.

One way to do so is sketched in Figure 1. Here is a non-exhaustive list of more guidelines:

(a) *Addressing the reader.* There are various options to write a text, examples are: *In this chapter, I discuss methods ...*, or *in this chapter, we discuss methods ....* Personally, I prefer the second option. Writing 'we' creates some familiarity and takes the reader by the hand. It is, however, entirely uncommon to address the reader by 'you' (in the sense of *building on these estimates, you get ...*).

(b) *Write out words.* Different from German, the English language offers to abbreviate some words (such as *can't, don't, won't, we'll* instead of *cannot, do not, will not, we will*). Stick to the second option and write out these words. Please do not confuse this with well-known abbreviations such as *e.g., i.e.,* and so forth.

(c) *Sentence length.* One sometimes tends to write long sentences, ranging over 4–5 lines (me included). Reading such sentences, it is often difficult to follow the precise argument, and confusion arises. Split the sentences into smaller pieces, still maintaining good readability of the text.

(d) *Mathematical statements: Give the precise assumptions.* Theorems, propositions, lemmas and corollaries *work subject to certain hypotheses.* You are usually required to precisely formulate these hypotheses – do so. Of course, it is unfavorable for the reader to gather the hypotheses under which a certain result holds on her own.

Sometimes it might happen that you have standing assumptions throughout a single chapter. In this case, make sure that the reader is well aware of these assumptions – by emphasizing them at the beginning of the chapter. One option to do so, e.g., is to include a text part such as: *Throughout the present chapter, we suppose that the function* $f\colon \mathbb{R} \to \mathbb{R}$ *satisfies the following set of assumptions: (...).*

(e) *Connect the underlying ideas.* When you state mathematical results, please do not follow the lecture-type style *Theorem – Proof – Corollary – Lemma – etc.*. Instead, make sure that the reader can follow you and understand why you proceed in the way you do.

We conclude this section with some remarks on mathematical typesetting:

(a) *Write formulas in the math mode.* Whenever you place a formula, use the Tex maths environments. This is usually displayed by the formulas appearing in italics. Here is an example:

Bad style: Let H be a Hilbert space and U be a closed subspace of H.

Better: Let H be a Hilbert space and U be a closed subspace of H.

(b) *Certain formulas should be non-italic.* Certain formulas are usually typeset with upright letters, so for instance the 'd' in differentials (e.g., $\int f \,\mathrm{d}x$) or the divergence $\mathrm{div}(f)$ (instead of $div(f)$). You can achieve this by defining *commands* - have a look here:

https://www.overleaf.com/learn/latex/Commands.

(c) *Emphasize long formulas.* Sometimes formulas can become quite long. In this situation, it is often favorable to put such long formulas in an own line. An example:

Bad style: *By Young's inequality we obtain* $\int_{\mathbb{R}^n} |uv| \,\mathrm{d}x \leq \frac{1}{2} \int_{\mathbb{R}^n} |u|^2 \,\mathrm{d}x + \frac{1}{2} \int_{\mathbb{R}^n} |v|^2 \,\mathrm{d}x$ *for all* $u, v \in \mathrm{L}^2(\mathbb{R}^n)$. *More generally, we obtain* $\int_{\mathbb{R}^n} |uv| \,\mathrm{d}x \leq \frac{1}{p} \int_{\mathbb{R}^n} |u|^p \,\mathrm{d}x + \frac{1}{p'} \int_{\mathbb{R}^n} |v|^{p'} \,\mathrm{d}x$ *for all* $u \in \mathrm{L}^p(\mathbb{R}^n)$ *and* $v \in \mathrm{L}^{p'}(\mathbb{R}^n)$.

Better: *By Young's inequality we obtain*

$$\int_{\mathbb{R}^n} |uv| \,\mathrm{d}x \leq \frac{1}{2} \int_{\mathbb{R}^n} |u|^2 \,\mathrm{d}x + \frac{1}{2} \int_{\mathbb{R}^n} |v|^2 \,\mathrm{d}x$$

for all $u, v \in \mathrm{L}^2(\mathbb{R}^n)$. *More generally, we obtain*

$$\int_{\mathbb{R}^n} |uv| \,\mathrm{d}x \leq \frac{1}{p} \int_{\mathbb{R}^n} |u|^p \,\mathrm{d}x + \frac{1}{p'} \int_{\mathbb{R}^n} |v|^{p'} \,\mathrm{d}x$$

for all $u \in \mathrm{L}^p(\mathbb{R}^n)$ *and* $v \in \mathrm{L}^{p'}(\mathbb{R}^n)$.

(d) *Write full sentences.* When writing a mathematical text, give full sentences. Here is an example:

Bad style: $ab \leq \frac{1}{2}a^2 + \frac{1}{2}b^2$ *for all* $a, b \in \mathbb{R}$, *therefore*

$$\int_{\mathbb{R}^n} |uv| \,\mathrm{d}x \leq \frac{1}{2} \int_{\mathbb{R}^n} |u|^2 \,\mathrm{d}x + \frac{1}{2} \int_{\mathbb{R}^n} |v|^2 \,\mathrm{d}x$$

for all $u, v \in L^2(\mathbb{R}^n)$.

Better: *Since for all real numbers* $a, b \in \mathbb{R}$ *there holds* $ab \leq \frac{1}{2}a^2 + \frac{1}{2}b^2$, *we conclude that*

$$\int_{\mathbb{R}^n} |uv| \,\mathrm{d}x \leq \frac{1}{2} \int_{\mathbb{R}^n} |u|^2 \,\mathrm{d}x + \frac{1}{2} \int_{\mathbb{R}^n} |v|^2 \,\mathrm{d}x$$

holds for all $u, v \in L^2(\mathbb{R}^n)$.

(e) *Conclude sentences with a dot.* Moreover, every sentence is concluded with a dot '.' – this principle equally applies to sentences which end with a formula. For instance, continuing with a slight modification of the above example:

Bad style: *Let* $u, v \in L^2(\mathbb{R}^n)$. *Since for all real numbers* $a, b \in \mathbb{R}$ *there holds* $ab \leq \frac{1}{2}a^2 + \frac{1}{2}b^2$, *we conclude that*

$$\int_{\mathbb{R}^n} |uv| \, \mathrm{d}x \leq \frac{1}{2} \int_{\mathbb{R}^n} |u|^2 \, \mathrm{d}x + \frac{1}{2} \int_{\mathbb{R}^n} |v|^2 \, \mathrm{d}x$$

This is a variant of Young's inequality.

Better: *Let* $u, v \in L^2(\mathbb{R}^n)$. *Since for all real numbers* $a, b \in \mathbb{R}$ *there holds* $ab \leq \frac{1}{2}a^2 + \frac{1}{2}b^2$, *we conclude that*

$$\int_{\mathbb{R}^n} |uv| \, \mathrm{d}x \leq \frac{1}{2} \int_{\mathbb{R}^n} |u|^2 \, \mathrm{d}x + \frac{1}{2} \int_{\mathbb{R}^n} |v|^2 \, \mathrm{d}x.$$

This is a variant of Young's inequality.

5 Referencing

Mathematical referencing works somewhat differently from other sciences. One of the key points is that we rarely use footnotes. Instead, one uses BibTex and the 'cite'-environment – see here for an introduction:

https://de.overleaf.com/learn/latex/Bibliography_management_with_bibtex.

A final point: Refer to books or published papers, and do not refer to certain unpublished lecture notes. Here, the idea is that your references should be accessible – if, e.g., informal lecture notes are removed from the web, then the particular reference becomes quite useless. This, of course, cannot happen with published references (or papers which are available through the ArXiv website, for instance).